Thermoelastic Damping of Vibrations in a Transversely Isotropic Hollow Cylinder

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***Abstract*-The purpose of the paper is to analyze the damping of three-dimensional free vibrations in a transversely isotropic, thermoelastic hollow cylinder, which is initially undeformed and kept at uniform temperature. The surfaces of the cylinder are subjected to stress free and thermally insulated boundary conditions. The displacement potential functions have been introduced for decoupling the purely shear and longitudinal motions in the equations of motion and heat equation. The purely transverse wave gets decoupled from rest of the motion and is not affected by thermal field. By using the method of separation of variables, the system of governing partial differential equations is reduced to four second order coupled ordinary differential equation in radial coordinate. The matrix Frobenius method of extended power series is employed to obtain the solution of coupled ordinary differential equations along the radial coordinate. In order to illustrate the analytic results, the numerical solution of various relations and equations are carried out to compute lowest frequency and thermoelastic damping factor with MATLAB software programming for zinc material. The computer simulated results have been presented graphically.**

***Key words*: Damping; Frobenius method; Cylinder**

I. Introduction

The vibrations in thermoelastic materials have many applications in various fields of science and technology, namely aerospace, atomic physics, thermal power plants, chemical pipes, pressure vessels, offshore, submarine structure, civil engineering structure etc. The hollow cylinders are frequently used as structural components and their vibrations are obviously important for practical design. The investigations of wave propagation in different cylinderical structures have been carried out by many researchers [1-6]. Ponnusamy [7] studied wave propagation in a generalized thermoelastic solid cylinder of arbitrary cross section. Suhubi and Erbey [8] investigated longitudinal wave propagation in thermoelastic cylinder. Sharma [9] investigated the vibrations in a thermoelastic cylindrical panel with voids.

The theory of elastic vibrations and waves well established; see Graff [10] and Love [11].The objective of the present paper is to study the three dimensional vibration analysis of simply supported, homogeneous transversely isotropic, hollow cylinder of length 'L' and radius 'R'. Three displacement potential functions are employed for solving the equation of motion and heat equation. The purely transverse wavesget decoupled from the rest of motion and are not affected by thermal field. By using the method of separation of variables the model of instant vibration problem is reduced to four second order coupled ordinary differential equations in radial coordinates. One of the standard techniques to solve ordinary differential equations with variable coefficients is the Frobenius method available in literature, see Tomentschger [12]. The secular equation which governs the three dimensional vibration of hollow cylinder has been derived by using Matrix Frobenius method. The numerical solution of secular equation has been carried out by MATLAB programming to compute lowest frequency and thermoelastic damping factor which have been presented graphically with respect to the parameter t_L for first two modes of vibrations $(n = 1,2)$.

II. Formulation of Problem

We consider a homogeneous transversely isotropic, thermal conducting elastic hollow cylinder of length L and radius R at uniform temperature T_0 in the undisturbed state initially. The basic governing equation of motion and heat conduction for three-dimensional linear coupled homogeneous and transversely isotropic thermoelastic cylinder in cylindrical co-ordinates (r, θ, z) system, in the absence of body force and heat source, are given by

$$\sigma_{rr,r} + \frac{1}{r}\sigma_{r\theta,r} + \sigma_{rz,z} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = \rho\frac{\partial^2 u_r}{\partial t^2}$$

$$\sigma_{r\theta,r} + \frac{1}{r}\sigma_{\theta\theta,\theta} + \sigma_{\theta z,z} + \frac{2\sigma_{r\theta}}{r} = \rho\frac{\partial^2 u_\theta}{\partial t^2}$$

$$\sigma_{rz,r} + \frac{1}{r}\sigma_{\theta z,\theta} + \sigma_{zz,z} + \frac{\sigma_{rz}}{r} = \rho\frac{\partial^2 u_z}{\partial t^2}$$

$$K_1\left(T_{,rr} + \frac{1}{r}T_{,r} + \frac{1}{r^2}T_{,\theta\theta}\right) + K_3 T_{,zz} - \rho C_e \dot{T} \tag{1}$$

$$= T_0\frac{\partial}{\partial t}\left[\beta_1\left(e_{rr} + e_{\theta\theta}\right) + \beta_3 e_{zz}\right]$$

where

$$\sigma_{rr} = c_{11}e_{rr} + c_{12}e_{\theta\theta} + c_{13}e_{zz} - \beta_1 T$$

$$\sigma_{\theta\theta} = c_{12}e_{rr} + c_{11}e_{\theta\theta} + c_{13}e_{zz} - \beta_1 T$$

$$\sigma_{zz} = c_{13}e_{rr} + c_{13}e_{\theta\theta} + c_{33}e_{zz} - \beta_3 T \tag{2}$$

$$\sigma_{r\theta} = 2c_{66}e_{r\theta}$$

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$$\sigma_{\theta z} = 2c_{44}e_{\theta z}$$
$$\sigma_{rz} = 2c_{44}e_{rz}$$
$$e_{rr} = \frac{\partial u_r}{\partial r}, e_{\theta\theta} = \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r}, e_{zz} = \frac{\partial u_z}{\partial z}$$
$$e_{\theta z} = \frac{1}{2}\left(\frac{\partial u_\theta}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial \theta}\right),$$
$$e_{r\theta} = \frac{1}{2}\left(\frac{1}{r}\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}\right),$$
$$e_{rz} = \frac{1}{2}\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right), c_{66} = \frac{c_{11} - c_{12}}{2}$$
$$\beta_1 = (c_{11} + c_{12})\alpha_1 + c_{13}\alpha_3$$
$$\beta_3 = 2c_{13}\alpha_1 + c_{33}\alpha_3 \qquad (3)$$

Here $u = (u_r, u_\theta, u_z)$ is the displacement vector; $T(r, \theta, z, t)$ is the temperature change; c_{11}, c_{12}, c_{13} and c_{44} are elastic constants; α_1, α_3 and K_1, K_3 are the coefficients of linear thermal expansion and thermal conductivities along and perpendicular to the axis of symmetry respectively; ρ and C_e are the mass density and specific heat at constant strain respectively, e_{ij} the strain tensor ; σ_{ij} the stress tensor. The comma notation is used for spatial-derivatives and superimposed dot denotes time derivatives. We introduce potential functions ψ, G, W as used by Sharma [5]

$$u_r = \frac{\psi_{,\theta}}{r} - G_{,r}, u_\theta = -\frac{G_{,\theta}}{r} - \psi_{,r}, u_z = W_{,z} \qquad (4)$$

Using equation (4) in equation (1) we find that G, W, ψ, T satisfies the equations:

$$\left(c_{11}\nabla_1^2 + c_{44}\frac{\partial^2}{\partial z^2} - \rho\frac{\partial^2}{\partial t^2}\right)G - (c_{13} + c_{44})\frac{\partial^2 W}{\partial z^2} + \beta_1 T = 0$$
$$-(c_{13} + c_{44})\nabla_1^2 G + \left(c_{44}\nabla_1^2 + c_{33}\frac{\partial^2}{\partial z^2} - \rho\frac{\partial^2}{\partial t^2}\right)W - \beta_3 T = 0 \qquad (5)$$
$$\left(K_1\nabla_1^2 + K_3\frac{\partial^2}{\partial z^2} - \rho c_e\frac{\partial}{\partial t}\right)T + T_0\frac{\partial}{\partial t}\left(\beta_1\nabla_1^2 G - \beta_3\frac{\partial^2 W}{\partial z^2}\right) = 0$$
$$\left(c_{66}\nabla_1^2 + c_{44}\frac{\partial^2}{\partial z^2} - \rho\frac{\partial^2}{\partial t^2}\right)\psi = 0 \qquad (6)$$

where

$$\nabla_1^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} \qquad (7)$$

We consider the free vibrations of a right circular hollow cylinder subject to traction free and thermally insulated or isothermal boundary conditions on the surface $r = R$ which is simply supported on the edges $z = 0$ and $z = L$. Therefore, we assume solution for three displacementfunctions and temperature change as

$$G(r,\theta,z,t) = \overline{G}(r)\sin(p\pi z)e^{i(n\theta-\omega t)}$$
$$W(r,\theta,z,t) = \overline{W}(r)\sin(p\pi z)e^{i(n\theta-\omega t)}$$
$$T(r,\theta,z,t) = \overline{T}(r)\sin(p\pi z)e^{i(n\theta-\omega t)} \qquad (8)$$
$$\psi(r,\theta,z,t) = \overline{\psi}(r)\sin(p\pi z)e^{i(n\theta-\omega t)}$$

On using solutions (8) in equations (5) and (6), we get

$$\left(\nabla_2^2 + k_1^2\right)\overline{\psi} = 0 \qquad (9.1)$$
$$\left(\nabla_2^2 + g_1\right)\overline{G} + g_2\overline{W} + \beta^*\overline{T} = 0 \qquad (9.2)$$
$$-c_3\nabla_2^2\overline{G} + c_2\left(\nabla_2^2 + g_3\right)\overline{W} - \overline{\beta}\beta^*\overline{T} = 0 \qquad (9.3)$$
$$a\left(\nabla_2^2\overline{G} + \overline{\beta}t_L^2\overline{W}\right) + \beta^*\left(\nabla_2^2 + g_4\right)\overline{T} = 0 \qquad (9.4)$$

Here we have used the following non-dimensional quantities

$$r' = \frac{r}{R}, z' = \frac{z}{L}, u_i' = \frac{u_i}{R}, T = \frac{T}{T_0}, R = \frac{(R_1 + R_2)}{2}, c_1 = \frac{c_{33}}{c_{11}},$$
$$c_2 = \frac{c_{44}}{c_{11}}, c_3 = \frac{c_{13} + c_{44}}{c_{11}}, c_4 = \frac{c_{66}}{c_{11}}, \overline{K} = \frac{K_3}{K_1}, \overline{\beta} = \frac{\beta_3}{\beta_1}$$
$$\beta^* = \frac{\beta_1 T_0 R^2}{c_{11}}, \quad \varepsilon_1 = \frac{\beta_1^2 T_0}{\rho c_e c_{11}}, g_1 = c_2\left(\Omega^2 - t_L^2\right),$$
$$g_2 = c_3 t_L^2, g_3 = \Omega^2 - \frac{c_1}{c_2}t_L^2, \quad g_4 = \frac{ic_2}{\chi^*}\left(\Omega^2 - \frac{i\overline{K}t_L^2\chi^*}{c_2}\right)$$
$$k_1^2 = \frac{c_{44}\left(\Omega^2 - t_L^2\right)}{c_{66}}, v_2 = \sqrt{\frac{c_{44}}{\rho}}, \omega^* = \frac{c_e c_{11}}{K_1}, \Omega = \frac{\omega R}{v_2}$$
$$t_L = \frac{p\pi R}{L}, \quad \chi^* = \frac{\omega}{\omega^*}, \quad a = \frac{\varepsilon_1 c_2 \Omega^2}{i\chi^*}$$

where

$$\nabla_2^2 = \frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{n^2}{r^2} \qquad (10)$$

Here v_2 is the velocity of purely elastic wave in hollow cylinder and ω^* is the characteristic frequency. The equation (9.1) represents purely transverse wave, which is not affected by the temperature change. The equation (9.1) is a Bessel's equation and its possible solutions are

$$\overline{\psi}(r) = \begin{cases} E_7 J_n(k_1 r) + E_7' Y_n(k_1 r), & k_1^2 > 0 \\ E_7 r^n + E_7' r^{-n}, & k_1^2 = 0 \\ E_7 I_n(k_1' r) + E_7' K_n(k_1' r), & k_1^2 < 0 \end{cases} \qquad (11)$$

where $k_1'^2 = -k_1^2$. Here E_7 and E_7' are two arbitrary constants, and J_n and Y_n are Bessel functions for first and second kind and I_n and K_n are modified Bessel functions for the first and second kind respectively. Generally $k_1^2 \neq 0$, so we go on with our derivation by taking the form of $\overline{\psi}$ for $k_1^2 < 0$, the derivation for $k_1^2 > 0$ is obviously similar. Therefore the solution valid in case of hollow cylinder is taken here as

$$\overline{\psi}(r) = E_7 I_n(k_1' r) + E_7' K_n(k_1' r) \qquad (12)$$

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III. METHOD OF FROBENIUS

In order to solve the coupled system of differential equations (9.2)-(9.4) we shall use matrix Frobenius method. The differential equations (9.2), (9.3) and (9.4), can be written in matrix form as

$$A\nabla_2^2\overline{Z} = -B\overline{Z} \tag{13}$$

where

$$A = \begin{pmatrix} 1 & 0 & 0 \\ -c_3 & c_2 & 0 \\ a & 0 & \beta^* \end{pmatrix}$$

$$B = \begin{pmatrix} g_1 & g_2 & \beta^* \\ 0 & c_2 g_3 & -\bar{\beta}\beta^* \\ 0 & a\bar{\beta}t_L^2 & \beta^* g_4 \end{pmatrix}, \overline{Z} = \begin{pmatrix} \overline{G} \\ \overline{W} \\ \overline{T} \end{pmatrix}, Z = \begin{pmatrix} G \\ W \\ T \end{pmatrix} \tag{14}$$

A standard technique for solving ordinary differential equations is the method of Frobenius, in which the solutions are in the form of power series. Clearly $r = 0$ is a regular singular point of the matrix differential equations (13) and hence, we take the solution of type

$$\overline{Z}\ (s,r) = \sum_{k=0}^{\infty} Z_k(s) r^{s+k} \tag{15}$$

where

$$\overline{Z}(s,r) = \begin{pmatrix} \overline{G}(s,r) \\ \overline{W}(s,r) \\ \overline{T}(s,r) \end{pmatrix}, \quad Z_k(s) = \begin{pmatrix} L_k(s) \\ M_k(s) \\ N_k(s) \end{pmatrix}.$$

Here the coefficients $L_k(s), M_k(s), N_k(s)$ and the parameter s (real or complex) are to be determined. By using matrix frobenius method the solution (15) becomes

$$\overline{Z}\ (s,r) = \sum_{k=0}^{\infty} Z_{2k}(s) r^{s+2k} \tag{16}$$

where

$$Z_{2k}(s) = \frac{1}{\left[(s+2)^2 - n^2\right]\left[(s+4)^2 - n^2\right]\cdots\left[(s+2k)^2 - n^2\right]} C^k Z_0 \quad \forall k = 0,1,2\ldots$$

$$Z_{2k+1} = 0 \quad \forall \quad k = 0,1,2,3..$$

$$C = -(A^{-1}B) = \begin{pmatrix} -g_1 & -g_2 & -\beta^* \\ -\dfrac{g_1 c_3}{c_2} & -\dfrac{(c_3 g_2 + g_3 c_2)}{c_2} & \dfrac{(\bar{\beta} - c_3)\beta^*}{c_2} \\ \dfrac{a g_1}{\beta^*} & \dfrac{a(g_2 - \bar{\beta} t_L^2)}{\beta^*} & (a - g_4) \end{pmatrix}$$

$$Z_0 = \begin{bmatrix} 1 & \dfrac{c_3}{c_2} & \dfrac{-a}{\beta^*} \end{bmatrix}^T L_0$$

Case-I (when the roots of indicial equation are distinct and do not differ by integer)

Thus the general solution of equation (10) has the form

$$\overline{Z}(r) = \sum_{i=1}^{3} E_i \overline{Z}_i(n,r) + \sum_{i=1}^{3} E_i' \overline{Z}_i(-n,r) \tag{17a}$$

where

$$\overline{Z}_i(s,r) = \frac{d^{i-1}}{ds^{i-1}}\left[r^s \sum_{k=0}^{\infty} Z_{2k}(s) r^{2k}\right]$$
$$\forall\ i = 1,2,3$$

Here E_1, E_2, E_3, E_1', E_2', E_3' are arbitrary constants to be evaluated by using boundary condition.

Hence

$$\{\overline{G}(r), \overline{W}(r), \overline{T}(r)\} = \sum_{i=1}^{3} E_i \{\overline{G}_i(n,r), \overline{W}_i(n,r), \overline{T}_i(n,r)\} + \sum_{i=1}^{3} E_i' \{\overline{G}_i(-n,r), \overline{W}_i(-n,r), \overline{T}_i(-n,r)\} \tag{18a}$$

where

$$\overline{G}_i(s,r) = \frac{d^{i-1}}{ds^{i-1}}\left[r^s \sum_{k=0}^{\infty} L_{2k}(s) r^{2k}\right] \quad \forall\ i = 1,2,3$$

$$\overline{W}_i(s,r) = \frac{d^{i-1}}{ds^{i-1}}\left[r^s \sum_{k=0}^{\infty} M_{2k}(s) r^{2k}\right] \quad \forall\ i = 1,2,3$$

$$\overline{T}_i(s,r) = \frac{d^{i-1}}{ds^{i-1}}\left[r^s \sum_{k=0}^{\infty} N_{2k}(s) r^{2k}\right] \quad \forall\ i = 1,2,3$$

Here, potential functions Z(r) written from (8) by using (17a) as

$$Z(r) = \begin{bmatrix} \sum_{i=1}^{3} E_i \overline{Z}_i(n,r) \\ + \sum_{i=1}^{3} E_i' \overline{Z}_i(-n,r) \end{bmatrix} \sin(p\pi z) e^{i(n\theta - \omega t)}$$

Here, potential function G, W, T written from (8) by using (18a) as

$$\{G(r), W(r), T(r)\} = \begin{Bmatrix} \sum_{i=1}^{3} E_i \{\overline{G}_i(n,r), \overline{W}_i(n,r), \overline{T}_i(n,r)\} \\ + \sum_{i=1}^{3} E_i' \{\overline{G}_i(-n,r), \overline{W}_i(-n,r), \overline{T}_i(-n,r)\} \end{Bmatrix} \sin(p\pi z) e^{i(n\theta - \omega t)} \tag{20a}$$

Case-II (when the roots of indicial equation are distinct and differ by integer)

Thus the general solution of equation (10) has the form

$$Z(r) = \sum_{i=1}^{6} E_i Z_i(n,r) \tag{17b}$$

where

$$\overline{Z}_i(s,r) = \frac{d^{i-1}}{ds^{i-1}}\left[r^s \sum_{k=0}^{\infty} Z_{2k}(s) r^{2k}\right]$$
$$\forall\ i = 1,2,3,4,5,6$$

Here E_1, E_2, E_3, E_4, E_5, E_6 are arbitrary constants to be evaluated by using boundary condition..

Hence

$$\{\overline{G}(r), \overline{W}(r), \overline{T}(r)\} = \sum_{i=1}^{6} E_i \{\overline{G}_i(n,r), \overline{W}_i(n,r), \overline{T}_i(n,r)\} \tag{18b}$$

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where

$$\bar{G}_i(s,r)=\frac{d^{i-1}}{ds^{i-1}}\left[r^s\sum_{k=0}^{\infty}L_{2k}(s)r^{2k}\right]\ \forall\ i=1,2,3,4,5,6$$

$$\bar{W}_i(s,r)=\frac{d^{i-1}}{ds^{i-1}}\left[r^s\sum_{k=0}^{\infty}M_{2k}(s)r^{2k}\right]\ \forall\ i=1,2,3,4,5,6$$

$$\bar{T}_i(s,r)=\frac{d^{i-1}}{ds^{i-1}}\left[r^s\sum_{k=0}^{\infty}N_{2k}(s)r^{2k}\right]\ \forall\ i=1,2,3,4,5,6$$

Here, potential functions Z(r) written from (8) by using (17b) as

$$Z(r)=\left[\sum_{i=1}^{6}E_i\bar{Z}_i(n,r)\right]\sin(p\pi z)e^{i(n\theta-\omega t)} \qquad (19b)$$

Here, potential function G, W, T written from (8) by using (31b) as

$$\{G(r),W(r),T(r)\}=\left\{\sum_{i=1}^{6}E_i\{\bar{G}_i(n,r),\bar{W}_i(n,r),\bar{T}_i(n,r)\}\right\}\sin(p\pi z)e^{i(n\theta-\omega t)} \qquad (20b)$$

IV. BOUNDARY CONDITIONS

We consider the free vibration of hollow–cylinder which is subjected to two types of boundary condition at lower and upper surface $(r = R_1, R_2)$

(A) *Mechanical condition*

The surface are assumed to be traction free, so that

$$\sigma_{rr}=\sigma_{r\theta}=\sigma_{rz}=0 \qquad (21)$$

(B) Thermal condition

The surface are assumed to be thermally insulated which leads to

$$T_{,r} = 0 \qquad (22)$$

V. FREQUENCY EQUATION

In this section we derive secular equation for thermoelastic hollow cylinder, subjected to traction free, and thermally insulated / isothermal boundary conditions at lower and upper surface $(r = R_1, R_2)$.The displacement, temperature change and stresses are obtained as

$$u_r=\left(-\bar{G}'+\frac{in}{r}\bar{\psi}\right)\sin(p\pi z)e^{i(n\theta-\omega t)}$$

$$u_\theta=\left(-\frac{in}{r}\bar{G}-\bar{\psi}'\right)\sin(p\pi z)e^{i(n\theta-\omega t)}$$

$$u_z=t_L\bar{W}\cos(p\pi z)e^{i(n\theta-\omega t)}$$

$$T=\bar{T}\sin(p\pi z)e^{i(n\theta-\omega t)} \qquad (23)$$

$$\sigma_{rr}=\begin{bmatrix}\frac{2c_4}{r}\bar{G}'+\left(g_1-\frac{2c_4n^2}{r^2}\right)\bar{G}\\ -\frac{2ic_4n}{r}\left(\bar{\psi}'-\frac{\bar{\psi}}{r}\right)+c_2t_L^2\bar{W}\end{bmatrix}\sin(p\pi z)e^{i(n\theta-\omega t)}$$

$$\sigma_{r\theta}=c_4\begin{bmatrix}-\frac{2ni}{r}\bar{G}'+\frac{2ni}{r^2}\bar{G}\\ -\bar{\psi}''-\frac{n^2}{r^2}\bar{\psi}+\frac{\bar{\psi}'}{r}\end{bmatrix}\sin(p\pi z)e^{i(n\theta-\omega t)}$$

$$\sigma_{rz}=c_2t_L\begin{bmatrix}-\bar{G}'+\\ \frac{in}{r}\bar{\psi}+\bar{W}'\end{bmatrix}\cos(p\pi z)e^{i(n\theta-\omega t)} \qquad (24)$$

where $\sigma'_{ri}=\frac{\sigma_{ri}}{c_{11}}$

prime has been suppressed for convenience.

Case: I-Using the equation (18a) in (24), we obtain the temperature gradient and stresses as

$$\sigma_{rr}=\begin{Bmatrix}\sum_{i=1}^{3}\{E_iF_i+E_i'F_i'\}\\ +E_7F_7+E_7'F_7'\end{Bmatrix}\sin(p\pi z)e^{i(n\theta-\omega t)}$$

$$\sigma_{r\theta}=\begin{Bmatrix}\sum_{i=1}^{3}\{E_iF_i^{*}+E_i'F_i^{*'}\}\\ +E_7F_7^{*}+E_7'F_7^{*'}\end{Bmatrix}\sin(p\pi z)e^{i(n\theta-\omega t)}$$

$$\sigma_{rz}=\begin{Bmatrix}\sum_{i=1}^{3}\{E_iF_i^{**}+E_i'F_i^{**'}\}\\ +E_7F_7+E_7'F_7^{**'}\end{Bmatrix}\cos(p\pi z)e^{i(n\theta-\omega t)}$$

$$T_{,r}=\sum_{i=1}^{6}\left\{E_iF_i^{***}+E_iF_i^{***'}\right\} \qquad (25a)$$

where

$$F_i=\begin{bmatrix}\frac{2c_4}{r}\bar{G}_i'(n,r)+\\ \left(g_1-\frac{2c_4n^2}{r^2}\right)\bar{G}_i(n,r)+c_2t_L^2\bar{W}_i(n,r)\end{bmatrix}$$

$$F_i'=\begin{bmatrix}\frac{2c_4}{r}\bar{G}_i'(-n,r)+\\ \left(g_1-\frac{2c_4n^2}{r^2}\right)\bar{G}_i(-n,r)+c_2t_L^2\bar{W}_i(-n,r)\end{bmatrix}$$

$$F_7=\left[-\frac{2ic_4nk_1'}{r}I_n'(k_1'r)\right]+\left[\frac{2ic_4n}{r^2}I_n(k_1'r)\right]$$

$$F_7'=\left[-\frac{2ic_4nk_1'}{r}K_n'(k_1'r)\right]+\left[\frac{2ic_4n}{r^2}K_n(k_1'r)\right]$$

$$F_i^{*}=\left[-\frac{2ni}{r}\bar{G}_i'(n,r)+\frac{2ni}{r^2}\bar{G}_i(n,r)\right]]$$

$$F_i^{*'}=\left[-\frac{2ni}{r}\bar{G}_i'(-n,r)+\frac{2ni}{r^2}\bar{G}_i(-n,r)\right]$$

$$F_7^{*}=\left[-k_1'^2I_n''(k_1'r)+\frac{k_1'}{r}I_n'(k_1'r)-\frac{n^2}{r^2}I_n(k_1'r)\right]$$

$$F_7^{*'} = \left[-k_1'^2 K_n''(k_1'r) + \frac{k_1'}{r} K_n'(k_1'r) - \frac{n^2}{r^2} K_n(k_1'r) \right]$$
$$F_i^{**} = \left[-\overline{G}_i'(n,r) + \overline{W}_i'(n,r) \right]$$

$$F_i^{**'} = \left[-\overline{G}_i'(-n,r) + \overline{W}_i'(-n,r) \right]$$
$$F_7^{**} = \left[\frac{in}{r^2} I_n(k_1'r) \right]$$
$$F_7^{**'} = \left[\frac{in}{r^2} K_n(k_1'r) \right]$$
$$F_i^{***} = \overline{T}_i'(n,r)$$
$$F_i^{***'} = \overline{T}_i'(-n,r)$$

Using the boundary conditions (21)-(22) and (25a), we get systems of eight simultaneously equations in $E_1, E_1', E_2, E_2', E_3, E_3', E_7, E_7'$, which will have non-trivial solution if the determinant of their co efficient vanishes. This requirement of non trivial solution leads to secular equation for hollow cylinder. The secular equation are obtained as

$$\left| p_{ij}' \right| = 0 \quad \forall i,j = 1,2,3,4,5,6,7,8 \qquad (26a)$$
$$p_{1j}' = \frac{2c_4}{t_1} \overline{G}_j'(n,t_1) + \left(g_1 - \frac{2c_4 n^2}{t_1^2} \right) \overline{G}_j(n,t_1) + c_2 t_1^2 \overline{W}_j(n,t_1) \quad \forall\ j = 1,2,3$$

$$p_{1j}' = \frac{2c_4}{t_1} \overline{G}_j'(-n,t_1) + \left(g_1 - \frac{2c_4 n^2}{t_1^2} \right) \overline{G}_j(-n,t_1) + c_2 t_1^2 \overline{W}_j(-n,t_1) \quad \forall\ j = 4,5,6$$
$$p_{17}' = -\frac{2ic_4 n k_1'}{t_1} I_n'(k_1' t_1) + \frac{2ic_4 n}{t_1^2} I_n(k_1' t_1)$$
$$p_{18}' = -\frac{2ic_4 n k_1'}{t_1} K_n'(k_1' t_1) + \frac{2ic_4 n}{t_1^2} K_n(k_1' t_1)$$
$$p_{2j}' = -\frac{2in}{t_1} \overline{G}_j'(n,t_1) + \frac{2in}{t_1^2} \overline{G}_j(n,t_1) \quad \forall\ j = 1,2,3$$
$$p_{2j}' = -\frac{2in}{t_1} \overline{G}_j'(-n,t_1) + \frac{2in}{t_1^2} \overline{G}_j(-n,t_1) \ \forall\ j = 4,5,6$$
$$p_{27}' = -k_1'^2 I_n''(k_1't_1) + \frac{k_1'}{t_1} I_n'(k_1't_1) - \frac{n^2}{t_1^2} I_n(k_1't_1)$$
$$p_{28}' = -k_1'^2 K_n''(k_1't_1) + \frac{k_1'}{t_1} K_n'(k_1't_1) - \frac{n^2}{t_1^2} K_n(k_1't_1)$$
$$p_{3j}' = -\overline{G}_j'(n,t_1) + \overline{W}_j'(n,t_1)$$

$$p_{3j}' = -\overline{G}_j'(-n,t_1) + \overline{W}_j'(-n,t_1)$$
$$p_{37}' = \frac{in}{t_1} I_n(k_1't_1)$$
$$p_{38}' = \frac{in}{t_1} K_n(k_1't_1)$$
$$p_{4j}' = \overline{T}_j'(n,t_1)$$
$$p_{4j}' = \overline{T}_j'(-n,t_1)$$
$$p_{47}' = 0\ , \ p_{48}' = 0$$

while p_{ij}' , $i = 5,6,7,8$ can be obtained by just replacing t_1 in p_{ij}' by t_2

where $t_1 = \frac{R_1}{R} = 1 - \frac{q}{R}$ and $t_2 = \frac{R_2}{R} = 1 + \frac{q}{R}$ and $q = \frac{(R_2 - R_1)}{R}$ is the thickness to themean radius ratio to hollow cylinder

Case: II-Using the equation (18b) in (24), we obtained the temperature gradient and stresses as

$$\sigma_{rr} = \left\{ \begin{array}{l} \sum_{i=1}^{6} \{E_i F_i\} + \\ E_7 F_7 + E_7' F_7' \end{array} \right\} \sin(p\pi z) e^{i(n\theta - \omega t)}$$
$$\sigma_{r\theta} = \left\{ \begin{array}{l} \sum_{i=1}^{6} \{E_i F_i^*\} + \\ E_7 F_7 + E_7' F_7^{*'} \end{array} \right\} \sin(p\pi z) e^{i(n\theta - \omega t)}$$
$$\sigma_{rz} = \left\{ \begin{array}{l} \sum_{i=1}^{6} \{E_i F_i^{**}\} + \\ E_7 F_7 + E_7' F_7^{**'} \end{array} \right\} \cos(p\pi z) e^{i(n\theta - \omega t)}$$
$$T_{,r} = \sum_{i=1}^{6} \{E_i F_i^{***}\} \qquad (25b)$$

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where

$$F_i = \begin{bmatrix} \frac{2c_4}{r}\overline{G}_i'(n,r) + \\ \left(g_1 - \frac{2c_4 n^2}{r^2}\right)\overline{G}_i(n,r) + c_2 t_L^2 \overline{W}_i(n,r) \end{bmatrix}$$

$$F_7 = \left[-\frac{2ic_4 n k_1'}{r} I_n'(k_1' r)\right] + \left[\frac{2ic_4 n}{r^2} I_n(k_1' r)\right]$$

$$F_7' = \left[-\frac{2ic_4 n k_1'}{r} K_n'(k_1' r)\right] + \left[\frac{2ic_4 n}{r^2} K_n(k_1' r)\right]$$

$$F_i^* = \left[-\frac{2ni}{r}\overline{G}_i'(n,r) + \frac{2ni}{r^2}\overline{G}_i(n,r)\right]$$

$$F_7^* = \begin{bmatrix} -k_1'^2 I_n''(k_1' r) + \\ \frac{k_1'}{r} I_n'(k_1' r) - \frac{\beta^2}{r^2} I_n(k_1' r) \end{bmatrix}$$

$$F_7^{*'} = \begin{bmatrix} -k_1'^2 K_n''(k_1' r) + \\ \frac{k_1'}{r} K_n'(k_1' r) - \frac{\beta^2}{r^2} K_n(k_1' r) \end{bmatrix}$$

$$F_i^{**'} = \left[-\overline{G}_i'(-n,r) + \overline{W}_i'(-n,r)\right]$$

$$F_7^{**} = \left[\frac{in}{r^2} I_n(k_1' r)\right]$$

$$F_7^{**'} = \left[\frac{in}{r^2} K_n(k_1' r)\right]$$

$$F_i^{***} = \overline{T}_i'(n,r)$$

Using the boundary conditions (21)-(22) and (25b), we get systems of eight simultaneously equations in $E_1, E_2, E_3, E_4, E_5, E_6, E_7, E_7'$, which will have non-trivial solution if the determinant of their Co efficient vanishes. This requirement of non trivial solution leads to secular equation for hollow cylinder. The secular equation are obtained as

$$|p_{ij}| = 0 \quad \forall\, i,j = 1,2,3,4,5,6,7,8 \qquad (26b)$$

$$p_{1j} = \frac{2c_4}{t_1}\overline{G}_j'(n,t_1) + \left(g_1 - \frac{2c_4 n^2}{t_1^2}\right)\overline{G}_j(n,t_1) + c_2 t_L^2 \overline{W}_j(n,t_1) \quad \forall\, j = 1,2,3,4,5,6$$

$$p_{17} = -\frac{2ic_4 n k_1'}{t_1} I_n'(k_1' t_1) + \frac{2ic_4 n}{t_1^2} I_n(k_1' t_1)$$

$$p_{18} = -\frac{2ic_4 n k_1'}{t_1} K_n'(k_1' t_1) + \frac{2ic_4 n}{t_1^2} K_n(k_1' t_1)$$

$$p_{2j} = -\frac{2in}{t_1}\overline{G}_j'(n,t_1) + \frac{2in}{t_1^2}\overline{G}_j(n,t_1) \quad \forall\, j = 1,2,3,4,5,6$$

$$p_{27} = -k_1'^2 I_n''(k_1' t_1) + \frac{k_1'}{t_1} I_n'(k_1' t_1) - \frac{n^2}{t_1^2} I_n(k_1' t_1)$$

$$p_{28} = -k_1'^2 K_n''(k_1' t_1) + \frac{k_1'}{t_1} K_n'(k_1' t_1) - \frac{n^2}{t_1^2} K_n(k_1' t_1)$$

$$p_{3j} = -\overline{G}_j'(n,t_1) + \overline{W}_j'(n,t_1) \quad \forall\, j = 1,2,3,4,5,6$$

$$p_{37} = \frac{in}{t_1} I_n(k_1' t_1), \quad p_{38} = \frac{in}{t_1} K_n(k_1' t_1)$$

$$p_{4j} = \overline{T}_j'(n,t_1) \quad \forall\, j = 1,2,3,4,5,6$$

$$p_{47} = 0, \quad p_{48} = 0$$

while p_{ij}, $i = 5,6,7,8$ can be obtained by just replacing t_1 in p_{ij} by t_2 where $t_1 = \frac{R_1}{R} = 1 - \frac{q}{R}$ and $t_2 = \frac{R_2}{R} = 1 + \frac{q}{R}$ and $q = \frac{(R_2 - R_1)}{R}$ is the thickness to the mean radius ratio to hollow cylinder

VI. NUMERICAL RESULTS AND DISCUSSION

In order to illustrate and verify results obtained in previous sections, we present some numerical simulation results. For the purpose of numerical computation, we have considered zinc-crystal like material whose physical data is given below (Dhaliwal and Singh [14]).

$c_{11} = 1.628\times10^{11}\,Nm^{-2}$, $c_{12} = 0.362\times10^{11}\,Nm^{-2}$
$c_{13} = 0.508\times10^{11}\,Nm^{-2}$, $c_{33} = 0.627\times10^{11}\,Nm^{-2}$
$c_{44} = 0.770\times10^{11}\,Nm^{-2}$, $\beta_1 = 5.75\times10^{6}\,Nm^{-2}\deg^{-1}$
$\beta_3 = 5.75\times10^{6}\,Nm^{-2}\deg^{-1}$, $K_1 = 1.24\times10^{2}\,Wm^{-2}\deg^{-1}$
$K_3 = 1.24\times10^{2}\,Wm^{-2}\deg^{-1}$ $\omega^* = 5.01\times10^{11}\,s^{-1}$
$\rho = 7.14\times10^{3}\,kgm^{-3}$, $T_0 = 296K$

Due to presence of dissipation term in heat conduction equation, the frequency equation in general complex transcendental equation provides us complex value of frequency (ω). For fixed value of n and k, the lowest frequency (Ω) and dissipation factor (D) are defined as

$$\Omega = \omega_R R\left(\frac{\rho}{c_{44}}\right)^{\frac{1}{2}}, \qquad D = \omega_I R\left(\frac{\rho}{c_{44}}\right)^{\frac{1}{2}}$$

where $\omega_R = \mathrm{Re}(\omega)$ and $\omega_I = \mathrm{Im}(\omega)$.
The thermoelastic damping factor is given by

$$Q^{-1} = 2\left|\frac{\mathrm{Im}(\omega)}{\mathrm{Re}(\omega)}\right| = 2\left|\frac{\omega_I}{\omega_R}\right|$$

The numerical computation has been carried out for $n > 0$, $k > 0$ with the help of MATLAB files .The secular equation (26) has been expressed in the form of $\Omega = g(\Omega)$ and the fixed point iteration numerical technique as outlined in Sharma [15] is used to find approximate solution of

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$\Omega = g(\Omega)$ near to the initial guess of the root with tolerance (10^{-5}). We computed lowest frequency and thermoelastic damping factor of first two modes of vibrations $(n = 1, 2)$ for different value of parameter axial wave number t_L of the cylinder. The variations of computer simulated lowest frequency and thermoelastic damping factor are with parameter t_L in respect to first two modes of vibration in figures 1 to 2.Fig. 1 represents the variations of lowest frequency of first two modes of vibrations, versus axial wave number for different values of thickness to mean radius ratio of the simply supported hollow cylinder of zinc-crystal like material, respectively. It is observed that the lowest frequency increases monotonically with axial wave number in both first and second modes of vibrations though its magnitude has been noticed to be larger in second mode of vibrations as compared to that of first mode of vibrations. It is also noticed that the magnitude of lowest frequency for each modes of vibrationsincreases monotonically with increasing value of q.

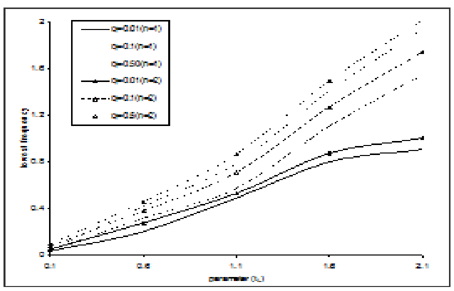

Fig 1. Lowest frequency (Ω) of first and second modes $(n = 1, 2)$versus parameter (t_L) for different values of thickness to mean radius ratio.

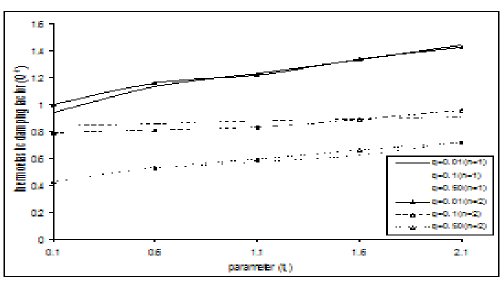

Fig 2. Thermoelastic damping factor (Q^{-1}) of first and second modes $(n = 1, 2)$versus parameter (t_L) for different values of thickness to mean radius ratio.

From fig. 2 it is observed that the thermoelastic damping factor of each mode of vibrationsincreases monotonically with increasing value of axial wave number for different values of thickness to mean radius ratio of the hollow cylinder. The trends of variations of thermoelastic damping factor for each considered mode of vibrations are almost steady and uniform for of thickness to mean radius ratio of the hollow cylinder. It is also revealed that the variations of thermoelastic damping factor for each mode of vibrations is dispersive in the range and remains close to each other for in case. Moreover, there exists atleast one value of parameterforeach at which the magnitude of thermoelastic damping factor is same for each considered mode of vibrations.

CONCLUSIONS

The modified Bessel functions and Matrix Frobenius method have been successfully used to study the vibrations of a homogeneous, transversely isotropic hollow cylinder based on three-dimensional thermoelasticity after decoupling the equations of motion and heat conduction with the use of potential functions. The decoupled purely transverse mode is found to be independent of rest of the motion and temperature change. The various thermal and mechanical parameters have significant effects on the natural frequency, thermoelastic damping factor of the hollow cylinder. The thermoelastic damping factor increases monotonically with axial wave number but decreases with thickness to mean radius ratio of the hollow cylinder.

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